Micro - 470

Mechanical Scaling Problem Set

Accelerometer design exercise

Problem 2: Design an accelerometer

- Your have maximum: 1 mm x 1 mm area (for cost reasons)
- Assume you are limited by thermal noise, and want to respond in 5 ms. Design an accelerometer: size of proof mass, and spring constant. Use Q=2
- What displacement would you have at the minimum detectable acceleration?
- Assuming your readout circuit can sense ΔC=5 aF, what is minimum number of fingers in readout capacitance (assuming single mask fabrication) to read a_{min}? Is this reasonable?
- Hints
  - as always, there is an infinite number of solutions: try to justify your choices from performance (sensitivity), fabrication feasibility, robustness, cost, etc.
  - Choose f_{res} to start the design
  - Use linear springs / assume linear behavior
  - How far would mass move at 10 g acceleration?

Accelerometer sensitivity and thermal noise

$$S_x = \frac{x}{a} = \frac{m}{k} \propto L^2$$

$$S_x = \frac{1}{\omega_0{}^2}$$

$$a_{\min} = \sqrt{\frac{4k_b T\omega_0}{mQ}}\sqrt{\Delta f}$$

Want low f_{res} for high sensitivity, ie high m, low k

If limit chip size, can only play with k (if we use full wafer thickness as the mass)

Want high m, and low f_{res} for low thermal noise

But

- low f_{res} means low bandwidth !
- Readout noise to be considered
- Mechanical robustness…

El Mansouri et al. *Microsystems & Nanoengineering* (2019)5:60
https://doi.org/10.1038/s41378-019-0105-y

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High-resolution MEMS inertial sensor combining large-displacement buckling behaviour with integrated capacitive readout

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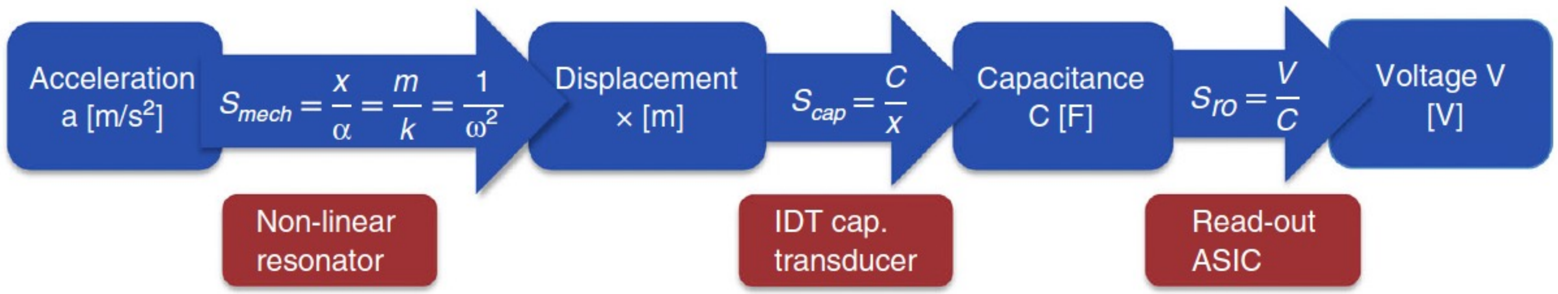

Fig. 1 System overview of the gravimeter; note the different sensitivities between every physical domain

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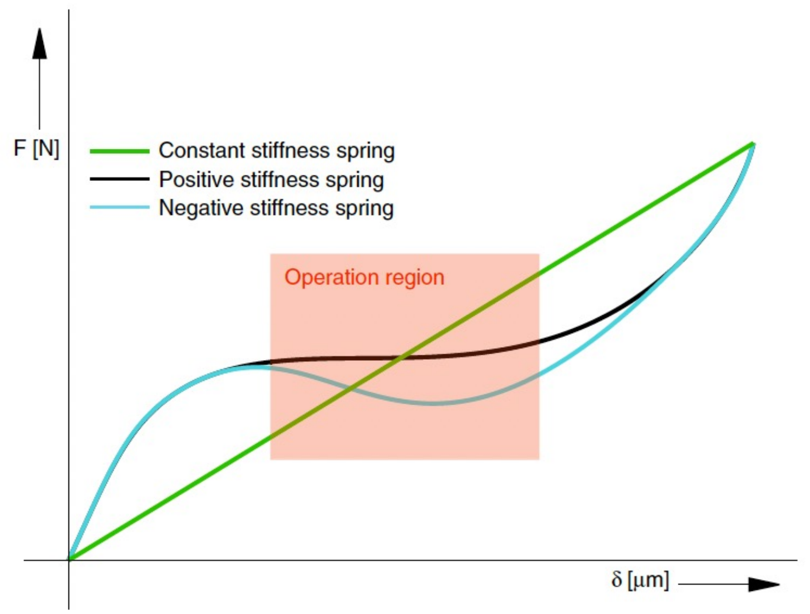
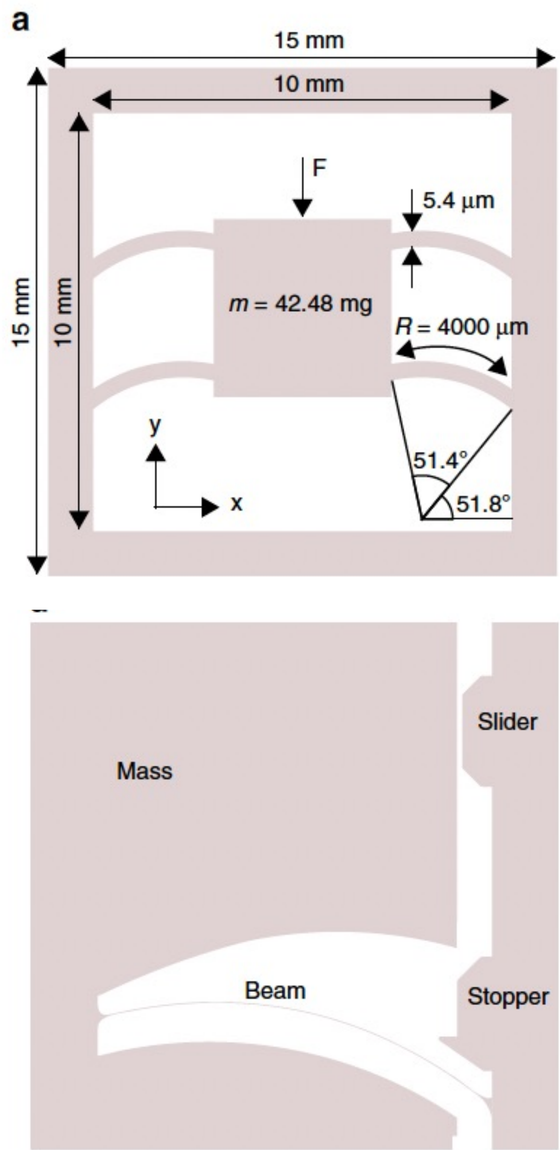


Fig. 2 Mechanical beam force/displacement behaviour showing: constant high positive stiffness (green), low local positive stiffness (black) and low local negative stiffness (blue)



They wanted really "soft" springs to have high sensitivity

The "trick" used by authors: buckling beams

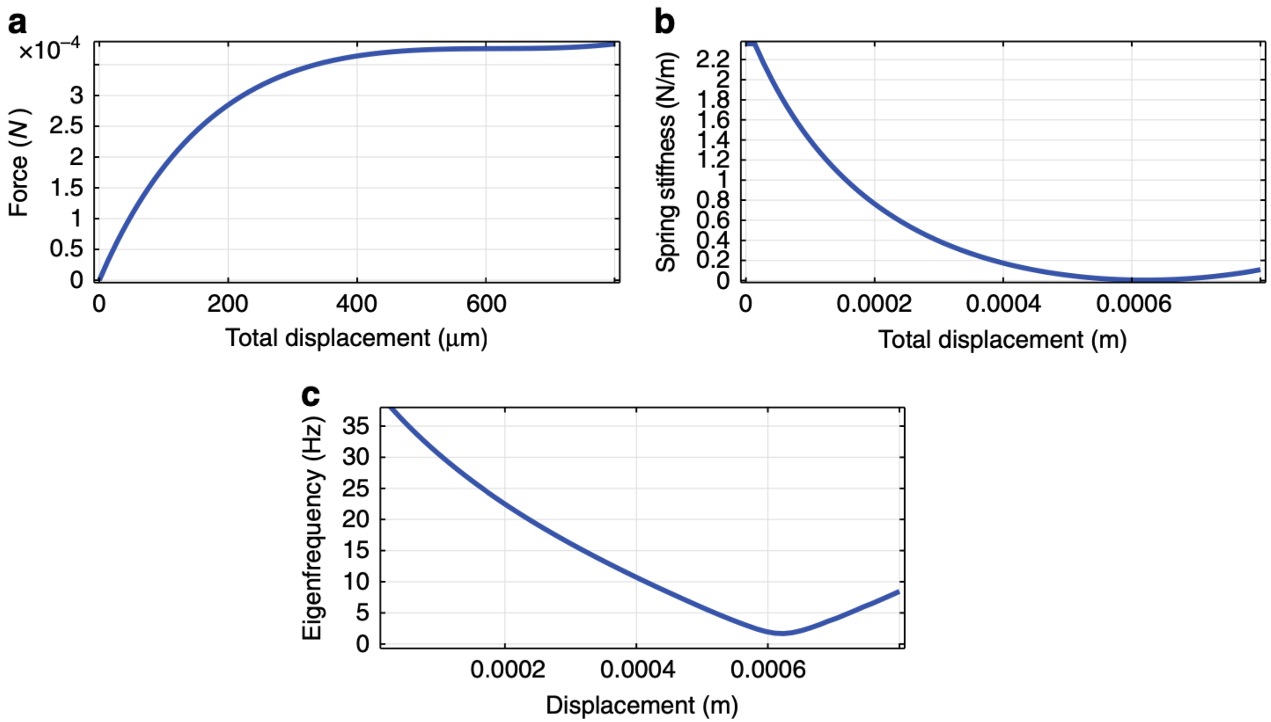
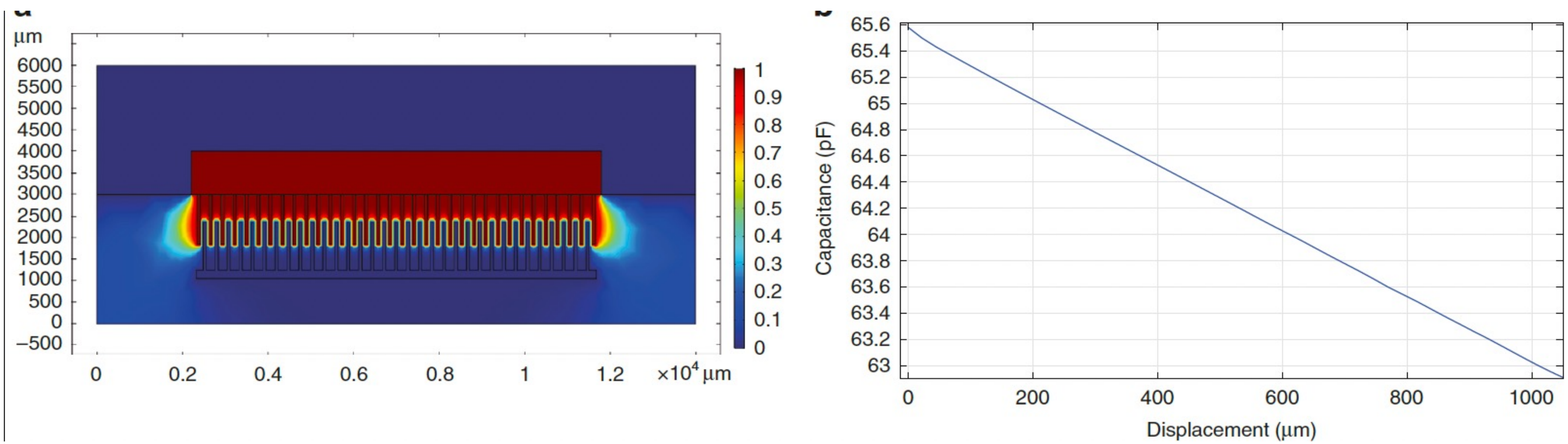
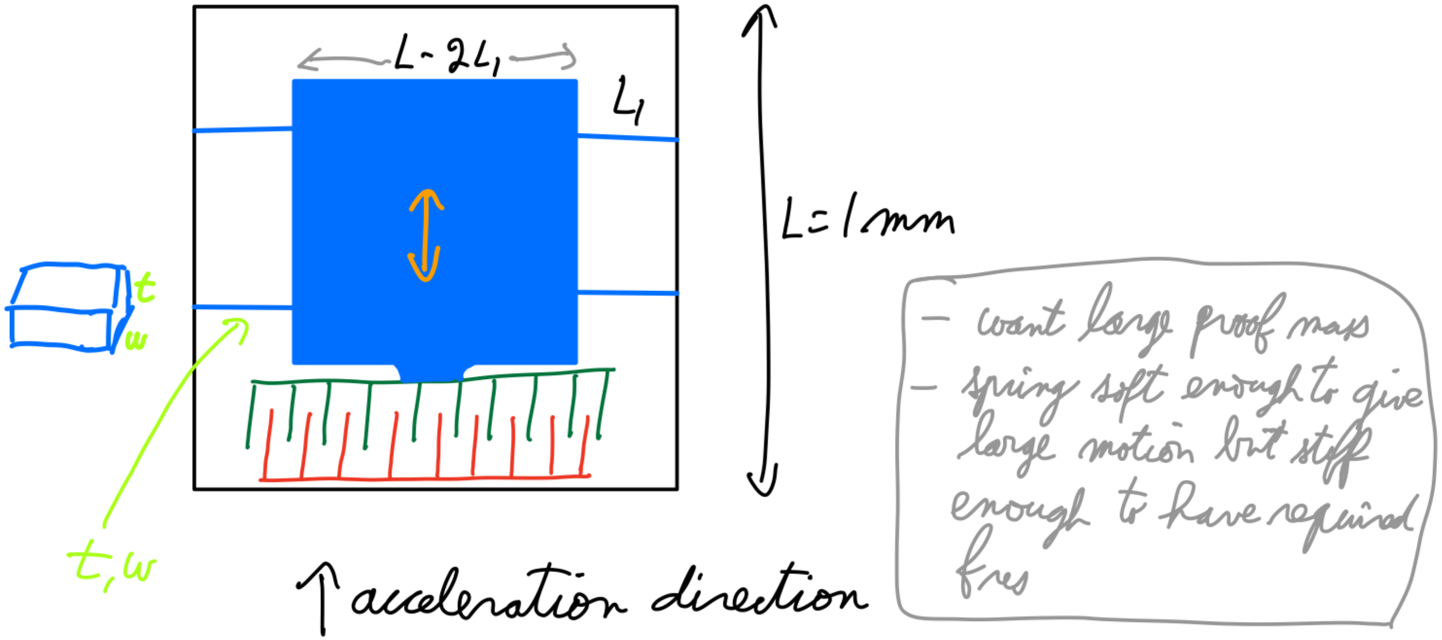


Fig. 4 FEM simulation results on the non-linear spring design. **a** Force vs. deflection of the proof mass in the *y* direction, **b** stiffness as a function of displacement in the *y* direction, **c** resonance (Eigen) frequency as a function of displacement



Readout noise from ASIC, within a BW of 20 Hz, is 0.137 aF/ sqrt(Hz)



1mm x 1 mm   5ms response   Q = 2

There are many possible solutions!

5ms response. I chose $f_{res} = 500\,Hz$ to work below resonance

$w_0 = 2\pi f = 2\pi \cdot 500 = 3000\,Hz = \sqrt{k/m}$

Choosing f_{res} set the sensitivity…

use w_0= 500 Hz to link spring and mass

Ie to find link between L_1 and w

– spring is simple beam, for simplicity

$$k = \frac{1}{4} E \frac{w^3 t}{L_1^3}$$ per spring. but 4 in parallel

$$m = \rho t (L - 2L_1)^2$$

ρ = density of Si

assume mass is same thickness as the springs (one mask process)

$$w_0^2 = \frac{E w^3 t}{L_1^3} \frac{1}{\rho t (L - 2L_1)^2}$$

$$\rightarrow w^3 = \frac{\rho w_0^2 L_1^3 (L - 2L_1)^2}{E}$$

no t dependence as $k \propto t$, $m \propto t$

eg $L_1 = 250\mu m$ $w = 0.8 \mu m$

$k = 0.012$ N/m per spring
$= 0.05$ N/m for 4.

· can make w wider, L longer higher w_0, at expense of smaller motion.

Noe can compute a_{min}

And hence min Δx

Choose t based on process, here assumes 20:1 etch ratio

$$a_{min} = \sqrt{\frac{4 k_B T \omega_0}{Q m}} \sqrt{\Delta f} \qquad \Delta f = 200\ Hz$$

thermal noise

for $t = 20\ \mu m$ $\quad a_{min} = 6.6 \cdot 10^{-4}\ m s^{-2}$ $\quad m = 1.2 \cdot 10^{-8}\ kg$

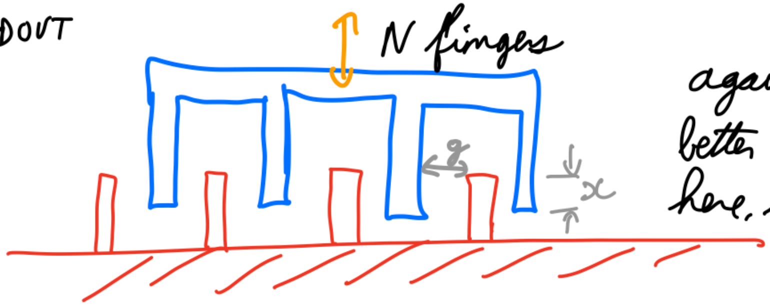
$T = 300 K$ $\quad = 66\ \mu G$

$$\Delta x \text{ for } a_{min} : \Delta x = \frac{m a}{k} = \frac{1.2 \cdot 10^{-8}\ 6.6 \cdot 10^{-4}}{0.05} = 1.5 \cdot 10^{-10}\ m$$

$$\Delta x_{MIN} = 0.15\ nm$$

- need to detect 0.15 nm but also e.g. 1G → $\Delta x = \frac{1.2 \cdot 10^{-8}\ 9.8}{0.05} = 2.3\ \mu m$
- would like thicker t for higher mass, but limited by etch process to t ~ 20.

READOUT



again, many options
better could be differential
here, simpler solution.

$C = \varepsilon_0 \frac{t x}{g}$

$\Delta C = \varepsilon_0 t \frac{\Delta x}{g}$ per finger

want g as small as possible.
but difficult to make
$g < t/20$

$\Delta C_{min} = \frac{\varepsilon_0 t \, \Delta x_{min}}{g} \cdot N$

$g = 1 \mu m \quad t = 20 \mu m$

$10^{-10} \cdot 20 \cdot 1.5 \cdot 10^{-10}$

$3 \cdot 10^{-20}$

$\Delta C_{min} = 3 \cdot 10^{-20}$ F per finger

readout circuit can sense $5aF = 5 \cdot 10^{-18} F$

$N \text{ fingers} = \frac{5 \cdot 10^{-18}}{3 \cdot 10^{-20}} = 167$ fingers

Was it fair to ignore the mass of the comb finger?